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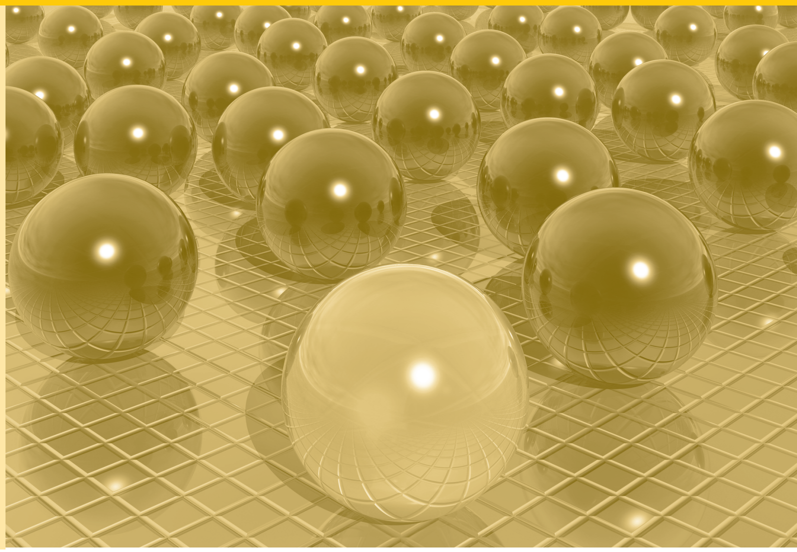

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How efficient are export champions?

Alexander Schiersch

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German engineering firms during the 1990's. How efficient are export champions?*

Alexander Schiersch†

Abstract

This paper investigates the efficiency of German engineering firms and its change over time. As these firms had been successful in the past in terms of being market-leader, it is expected that the majority of the firms works efficiently. To analyze this question Farrell's technical efficiency is estimated using DEA. The results contradict these expectations. The engineering firms proved to operate quite inefficiently. Moreover, the results indicate that the efficiency of the firms is almost normally distributed and differs with company sizes. Besides, the measured efficiency changes are not consistent with the expectation of essentially small changes over time.

JEL Classification: C14, L25, L64

Keywords: data envelopment analysis, technical efficiency, nonparametric, German engineering firms

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I. Introduction

The growth of the German economy depends to a great extent on the success of its exporting industries. Looking at the workforce employed in each sector, it is the engineering industry which is most vital for the German economy. This branch has expanded its output between 1996 and 2004 by roughly 15 percent. At the same time its export increased by more than 50 percent reaching a value of little less than 100 billion Euros. Moreover, German engineering firms have occupied leading positions in many foreign markets and were able to hold these positions (see e.g. Weiß, 2003, Wiechers and Schneider, 2005).

There are probably several reasons for this success. One of them is expected to be the efficiency of these firms. Therefore, this paper examines the technical efficiency[1] of Germanys engineering firms. According to the above mentioned prosperity it is expected to find a majority of the firms operating at an efficient level, i.e. with an efficiency score - scaled between zero and one – to be near one. This would fit with the basic economic proposition that in a competitive environment only efficient firms are successful and able to survive. Hence, the first hypothesis is: A majority of the German engineering firms is operating efficiently.

As there is always a minority of firms leaving the market it can also be expected to find a density of efficiency scores that is skewed to the left with a majority of firms operating quite efficiently while a minority does not. Moreover such skewness seems reasonable if one takes into account that the probability of survival ought to fall with increasing inefficiency and thus with efficiency scores below the mean. Hence, the second hypothesis is: The density of the efficiency of engineering firms is skeweed to the left. Besides efficiency, the study will also examine its changes over time. If the first hypothesis finds support, it would also seem reasonable to expect only small shares of changes in efficiency over time to be meaningful.

The remainder of this paper is organized as follows. Section II provides the reader with a short overview of the methodology. The third section is devoted to the description of the data and the employed variables. The results of the empirical analysis are presented in section IV whereas section V concludes.

II. Methodology

For the present analysis a nonparametric frontier approach was adopted to estimate Farrell's technical efficiency (Farrell, 1957). The fields of research that apply a nonparametric approach, especially the data envelopment analysis (DEA), are enormous. DEA was used for instance to evaluate the efficiency of the Jordanian Police Force or of regulated industries like the electricity distribution in Scandinavia. Tavares (2002) listed more than 3200 articles, working papers and books etc. that used DEA. The fundament for this success was created almost fifty years ago by the works of Koopmans, Debreu, Farrell and Shephard, to name the most known (Sei-

[1] Efficiency in the context of this paper is defined as technical efficiency. The technical efficiency measures the ability of the firm to produce the maximum output giving a set of inputs et vice versa (Farrell, 1957).

ford, 1996). Moreover, it has to be stated that Deprins et al. (1984) invented the practical approach to measure efficiency using a free disposal hull (FDH), while DEA became popular by the work of Charnes et al. (1978). To make the approaches actually applicable the statistical properties of their estimators are crucial. Here Banker (1993) and Kneip et al. (1998, 2003) as well as Simar and Wilson (2000b, 2002) have done the basic research.

There are several reasons for the popularity of nonparametric analysis in econometric research. One of them is the fact that no assumptions regarding the functional form of the production function are necessary.[2] Hence, the usual handicap of assuming a functional form out of theoretical considerations which may correspond with reality, but which could also be wrong, does not exist when applying nonparametric approaches. The actual measurement utilizes a best practice frontier based on actual observations. This frontier itself is defined by the production set

$$\Psi = \left\{(x, y) \in \mathbb{R}_+^{p+q} \middle| x\ can\ produce\ y\right\},$$

where $x \in \mathbb{R}_+^p$ contains a set of p inputs and $y \in \mathbb{R}_+^q$ contains a set of q outputs. Since an input orientated approach will be adopted the input requirement set for all $y \in \Psi$ is described as follows:

$$C(y) = \left\{x \in \mathbb{R}_+^p \middle| (x, y) \in \Psi\right\}.$$

Thus the frontier is finally defined by

$$\partial C(y) = \{x | x \in C(y), \theta x \notin C(y)\ \forall\ 0 < \theta < 1\}.$$

This frontier can be seen as an empirical production function derived out of real input/output combinations. The input oriented technical efficiency by Farrell for a DMU[3] with an observed combination of outputs and inputs (x_0, y_0) based on the frontier above is given by

$$\theta(x_0, y_0) = \inf\{\theta | \theta x_0 \in C(y_0)\} = \inf\{\theta | (\theta x_0, y_0) \in \Psi\},$$

whereas $\theta(x_0, y_0)$ is a radial measure, taking values between zero and one. Consequently, a company is considered to be efficient if it lies exactly on the frontier and hence its θ takes the value of one. In contrast, if $\theta(x_0, y_0)$ is below one (e.g. $\theta = 0.7$), the company would need to reduce the amount of assembled input by $1 - \theta$ percent in order to operate efficiently (e.g. a reduction by 30 percent or in other words $0.7 \times x_0$ to produce y_0 efficiently).

Of course researchers do not know the real production set. Typically there is just a sample of observations $\mathcal{X}_N = \{(x_i, y_i), i = 1, \dots, n\}$ and Ψ needs to be estimated. This can be done either by using the free disposal hull (FDH) method or by the data envelopment analysis (DEA). The present analysis is based on DEA. Depending on the assumed returns to scale, the production set has a convex or conical hull. The DEA estimator under variable returns to scale (and therefore with a convex hull) is defined as follows:

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[2] The assumptions that form the basis of these nonparametric models are rather rudimental, affecting mathematic questions like a closed input/output space etc. For further information on the assumptions see for instance Simar and Wilson (2005).

[3] DMU is the abbreviation for decision making unit.

$$\widehat{\Psi}_{VRS}(X_n) = \left\{(x,y) \in \mathbb{R}_+^{p+q} \middle| y \leq \sum_{i=1}^{n} \gamma_i y_i, x \geq \sum_{i=1}^{n} \gamma_i x_i, \sum_{i=1}^{n} \gamma_i = 1, \gamma_i \geq 0 \forall i = 1, \dots, n\right\},$$

and the technical efficiency can be calculated by solving the subsequent linear program

$$\hat{\theta}_{VRS} = min\{\theta > 0 | y \leq \sum_{i=1}^{n} \gamma_i y_i, \theta x \geq \sum_{i=1}^{n} \gamma_i x_i, \sum_{i=1}^{n} \gamma_i = 1, \gamma_i \geq 0 \forall i = 1, \dots, n\}.$$

However, a major concern in econometric analysis is the consistency of the estimator. Within this framework under the assumption of variable returns to scale (VRS) the estimator is always consistent, although he is not efficient if the production set exhibits constant (CRS) or non-increasing returns (NIRS) to scale (Simar and Wilson, 2002). On the other hand, if Ψ exhibits variable returns to scale and the model assumes constant or non-increasing returns to scale, both $\hat{\theta}_{CRS}$ and $\hat{\theta}_{NIRS}$ are inconsistent. Therefore two tests will be performed in order to examine the economies of scale of Ψ. Since the first one, the Kolmogorov-Smirnov test, is found to perform rather poorly within the DEA context by Simar and Wilson (2002), the Wilcoxon rank-sum test is also utilized to determine which assumption is appropriate.[4]

Another crucial point one has to look at is the potential bias of DEA – estimators. This bias traces back to the fact that the efficiency of a firm is measured by its position within the production set compared to the frontier of this production set. Since the latter is estimated itself based on a limited sample, $\widehat{\Psi}_{\mathrm{DEA}}$ is a subset of the unknown production set $\left(\widehat{\Psi}_{\mathrm{DEA}} \subseteq \Psi\right)$ by construction and therefore $\widehat{\partial C}_{DEA}(y)$ is inward – biased compared to $\partial C_{DEA}(y)$. Consequently the technical efficiency of a firm is potentially upward – biased or in other words too optimistic (Simar and Wilson, 2005). Hence, it is necessary to calculate bias-corrected estimates which can be done by bootstrapping. Unfortunately the naive bootstrap method leads to inconsistent estimates if applied to DEA-estimators since they are bounded (Simar and Wilson, 2005). Nevertheless, calculating bias corrected estimates is possible and was done in the present survey (see Simar and Wilson 1998, 2000a, 2000b and Kneip et al. 2003).

Besides the bias, nonparametric analyses are also “cursed” as some scientists put it. This “curse” is related to the convergence of estimators and the dimensionality. Within the DEA estimators converge at a rate of $n^{2/p+q+1}$. Thus, as the number of inputs and outputs applied in the calculation increases the rate of convergence decreases. A further problem appears with increasing dimensions. As shown by Wheelock and Wilson (2003) the number of efficient firms, i.e. of DMUs that lie on the frontier, increases with a rising number of dimensions. This is not surprising looking at the way the frontier is designed. It has to made sure therefore, that the number of defined inputs and outputs is limited.

Besides the actual efficiency its change over time is also of interest and therefore the Malmquist Index will be applied. In this context the essential works by Caves et al. (1982) and Färe et al. (1994) have to be mentioned.

[4] Both tests are nonparametric tests. The Kolmogorov-Smirnov test compares the distribution functions of two series postulating under H_0 that changes are just incidental. The reader is referred to Büning and Trenkler (1978) and Siegel and Castellan (1988) for more information. The Wilcoxon rank-sum test can be seen as a nonparametric alternative of the t-test. Here again the null hypothesis postulates identical distributed series. The reader is referred to Büning and Trenkler (1978) for more information on the actual construction of the test.

The idea of decomposing the index so that the efficiency change as well as changes in the frontier could be measured is frequently used since (Bjurek, 1996).

Now, decomposing the Malmquist Index is giving the *catching – up effect* (CE) as the required tool to evaluate efficiency changes. A CE above one reveals an improvement in efficiency between t and $t+1$.The opposite is true if the CE is less than one. As can be seen below, the CE also reveals the extend of change measured in percent.

$$CE = \frac{\theta_{t+1}(x_{t+1}, y_{t+1})}{\theta_t(x_t, y_t)}$$

Now, the equation above leads to the question whether one should use a common frontier when calculating θ_{t+1} and θ_t. Utilizing a common frontier implies a stable technology over the observed period. Without checking this is just an assumption nothing more. However, the impact of this assumption is as critical as the assumption concerning economies of scale. An erroneous assumption regarding the stability of technology could lead not only to false conclusions towards the efficiency and its change over time, the measured efficiency of firms will also be inconsistent.

Figure 1: Efficiency and time[5]

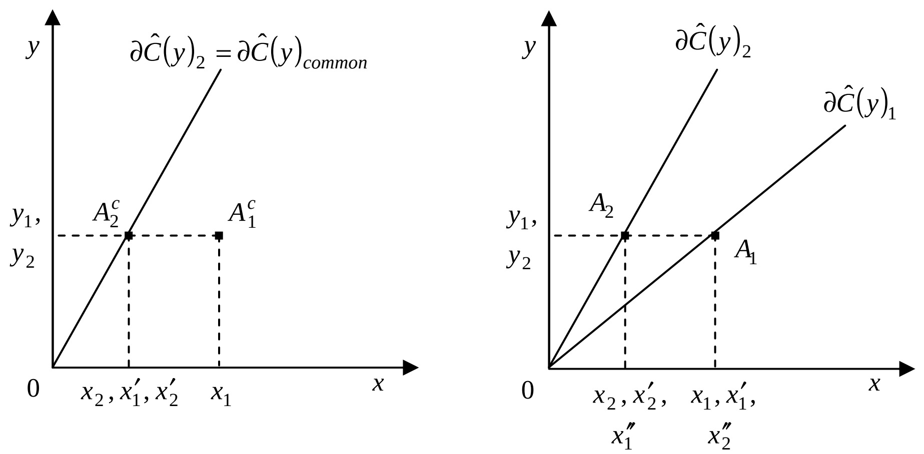


Figure 1a Figure 1b

The reason for this can be found again in the construction of the frontier. Consider a simple situation as presented in Figure 1 and suppose that company A works efficient ($\theta = 1$) at each specific point in time ($t, t+1$). So, if one presumes constant technology and therefore applies a common frontier, company A's input/output – combination in t will be measured as inefficient as shown in Figure 1a. On the other hand, checking for possible changes in technology and thus calculating two frontiers based on the observations of each time, the company

[5] The figure was taken from Grosskopf (1993) and adapted to the example above.

is found, as assumed, to be efficient at each specific point in time as shown in Figure 1b. Moreover, in the first case (Fig. 1a) company A seems to have enhanced its efficiency, while it actually was efficient the whole time (Fig. 1b). Hence, simply assuming a stable technology over time and applying a common frontier would result in inconsistent estimates. Therefore checking whether there was a change in technology or not is crucial.

A common tool to do this is the *frontier shift effect* (FSE), which is also a part of the malmquist index.

$$FSE = \left[\left(\frac{\theta_t(x_{t+1}, y_{t+1})}{\theta_{t+1}(x_{t+1}, y_{t+1})}\right)\left(\frac{\theta_t(x_t, y_t)}{\theta_{t+1}(x_t, y_t)}\right)\right]^{1/2}$$

Values larger than one indicate a positive development in technology. In this case, a company would need fewer inputs in $t+1$ producing the same output compared to the situation in t. The opposite is true if the FSE takes values below one. Then a company would have to incorporate more inputs in $t+1$ to produce the same amount of output as in t. Consequently a FSE of one shows that the technology was stable over time.

The explanation above is based on the assumption of constant returns to scale which is a certain limitation of the FSE. As already mentioned one has to check which assumption holds regarding the economies of scale. If the data reveals that the frontier is characterized by variable returns to scale, it is possible that frontiers intersect even in the two dimensional case. Moreover, intersecting frontiers are also possible under all scale assumptions if the dimension is above two.[6] In consequence the FSE can lose some of its explanatory power concerning the direction of a measured frontier shift and its extent.[7] This leads to the final question why the FSE should be used at all? Even with the disadvantage that easily appears the FSE can still only take values above or below one if there was a change in technology and consequently a frontier shift. Thus, it still provides the necessary information needed to decide whether or not a common frontier should be estimated.

In order to derive consistent estimates the calculations will have to follow a strict procedure. According to the theoretical remarks the first step is evaluating which scale assumption holds. As pointed out in this section a correct assumption is crucial in order to avoid inconsistent estimates. Hence, the actual analysis starts by estimating efficiency scores while utilizing each scale assumption and applying both the Kolmogorov-Smirnov test and the Wilcoxon rank-sum test. Doing this, a kind of circulus vitiosus occurs. In order to estimate consistent efficiency scores when testing the scale assumption, one should know if the technology was stable over time and therefore if a common frontier can be utilized or not. On the other hand, knowing the correct scale assumption is necessary for calculating consistent efficiency scores, which than can be used to determine the correct frontier approach.

Here, the first calculations were conducted applying a yearly frontier and utilizing each scale assumption. Thus, the possibility of inconsistent estimates due to a wrong assumption concerning the appropriate time frame applied when constructing the frontier is avoided. Finally, three series of efficiency scores are available, each one estimated under a different scale assumption. Based on these data the Kolmogorov-Smirnov test and the

[6] A comprehensible example for the three dimensional case under constant returns to scale can be found in Førsund (1993).

[7] See Appendix B.

Wilcoxon rank-sum test will be carried out determining the correct scale assumption. Giving this assumption the question whether there was a change in technology and thus a frontier shift can be answered calculating the FSE. Giving both assumptions the Hypotheses can be checked.

III. Data

This study uses data from the German Cost Structure Census of manufacturing (CSC) for the period 1995 to 2004. This sample is gathered by the German Federal Statistic office and firms are obliged to deliver the requested data. All companies with more than 500 employees are constantly part of the sample. In addition smaller companies are also included as a random subsample. The latter is held constant for four years before a new subsample is created. Hence, within a time frame of four years the sample structure ought to be stable.

Since it is the efficiency of engineering firms that is of interest, all other companies are excluded, giving us a sample of 23,591 observations between 1995 and 2004 with a wide range of characteristics.[8] This large set of information is generally appreciable but the chosen dimension should not be too high because of the already mentioned “curse”. Hence, in order to avoid implausible estimates the characteristics were summed up to create five factors. The first one is output, which is basically the gross production value, adjusted by all revenues that have nothing to do with the core business of an engineering firm, like turnover out of ‘other activities’ or ‘trading goods’. These revenues were excluded since they have no explanatory power in terms of a production function.

However, the model contains the input factors: ‘material’, ‘capital costs’, ‘cost of labor’ and ‘other costs’. The characteristic ‘material’ includes all materials and preliminary products as well as energy. The factor ‘capital costs’ contains first of all interest and amortization. Moreover, rental and leasing expenditures need to be taken into account. Since a company could also buy a machine or a building rather than rent it, ‘capital costs’ also include these expenses. The third input factor is also made up of several characteristics. It entails all wages including the social insurance contributions that have to be paid by the companies. In addition all social benefits (e.g. employee pensions) guaranteed by firms beyond legal requirements are summed up in the factor. The remaining costs like repair, installation etc. are put into the factor ‘other cost’. After creating these new factors their values are deflated.

Besides merging and deflating, the data were also adjusted for possible outliers and missing values. This is crucial since nonparametric frontier models are sensitive to both, due to the fact that they do not assume any functional form of the production function. Thus, all companies without values in one of the created input factors have been excluded. Thereafter each input factor was divided by its corresponding output (that is the in- and output for each firm) and all observations with a value greater than the 99.5 percentile are also excluded. Finally all companies with just one observation over the whole period are deleted, as they are useless in terms of changes in efficiency. Thus, the final sample contains 5,464 companies with 20,589 observations which are

[8] Almost all characteristics entail information in monetary form. In this study only the monetary informations are used and summed up. The only exception is the number of employees, but these are not used as input factor in the calculation.

quite constantly distributed over the years.[9] However, as Table 1 and 2 reveals about 50 percent of the companies are part of the sample with just two or three observations or two or three observations in successive years, respectively.[10]

Table 1: Share of firm-year observations

Years in the sample	share of all firms	cumulative share of all firms
2	42.48	42.48
3	8.25	50.73
4	24.2	74.93
5	4.03	78.96
6	12.23	91.19
7	1.26	92.45
8	3.44	95.89
9	0.71	96.6
10	3.4	100

Table 2: Share of consecutive observations

consecutive observations	share of all firms	cumulative share of all firms
1	2.47	2.47
2	45.02	47.50
3	4.90	52.39
4	24.19	76.58
5	1.68	78.26
6	6.59	84.84
7	1.26	86.10
8	4.08	90.18
9	0.79	90.97
10	9.03	100.00

IV. Results

According to the outlined proceeding the first step clarifies which scale assumption holds. The results of both tests are presented in Table 3. First of all it is obvious that the frontier does not exhibit constant returns to scale. The answer is not that simple if it comes to VRS versus NIRS. As can be seen in column three, in three years out of ten the null hypothesis cannot be rejected under the Kolmogorov-Smirnov test.[11] Hence, the frontier seems to exhibit non-increasing returns to scale in these years. But as pointed out earlier the results have to be confirmed by the Wilcoxon rank-sum test. As column four reveals, the null hypothesis can now be rejected in two of the three cases at a significance level of 10% or in one of the three years at a significance level of 5%, respectively.[12]

[9] See Table 8 in Appendix A.

[10] As mentioned further above, for each company within the sample there ought to be at least 4 observations. Unfortunately that is not always the case as the results in Table 1 shows. This could partly be explained by the fact that some companies exit the market. Moreover the CSC gives no information about mergers. So, if two companies merge the sample contains at least for one company with its unique ID no more observations in the following years. In addition in the first years of the CSC the rotating subsamples wasn't held constant.

[11] The null hypothesis assumes that the distribution of efficiency scores calculated under VRS and NIRS is identical. This only can happen if the frontiers under VRS and under NIRS would lie at the same coordinates in the $p + q$ dimensional space. This in turn indicates that the production function exhibits non-increasing returns to scale.

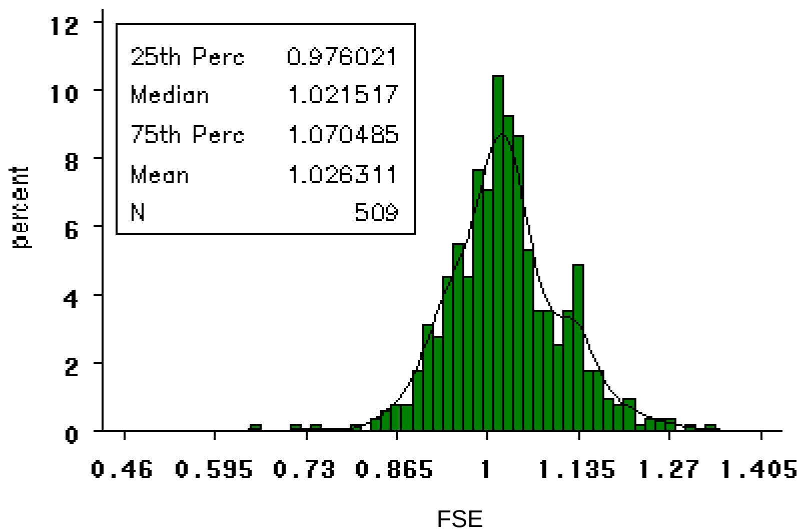
[12] See explanation above.

Table 3: Test for economics of scale

year	VRS vs. CRS p-value, KS-Test	VRS vs. CRS p-value, W-Test	VRS vs. NIRS p-value, KS-Test	VRS vs. NIRS p-value, W-Test
1995	<0.01	<0.01	<0.01	<0.01
1996	<0.01	<0.01	<0.01	<0.01
1997	<0.01	<0.01	0.5690	0.1476
1998	<0.01	<0.01	0.3035	0.0541
1999	<0.01	<0.01	<0.01	<0.01
2000	<0.01	<0.01	<0.01	<0.01
2001	<0.01	<0.01	0.01	<0.01
2002	<0.01	<0.01	0.1048	0.01
2003	<0.01	<0.01	<0.01	<0.01
2004	<0.01	<0.01	<0.01	<0.01

Consequently in at least two years the frontier exhibit non-increasing returns to scale. This leads to the question which assumption should be applied. In the following analysis variable returns to scale were assumed for the following reason: In at least seven out of ten years the null hypothesis was rejected. Appling NIRS could lead to inconsistent estimates in years where VRS was observed. If instead we assume VRS the estimates are definitely consistent even in years where the tests have found frontiers with non-increasing returns to scale.

Figure 2: The FSE based at 1995 and 2004[13]



Having determined the returns to scale assumption, that is VRS, the next step is answering the question whether or not a common frontier should be estimated by calculating the FSE. Since there are a number of

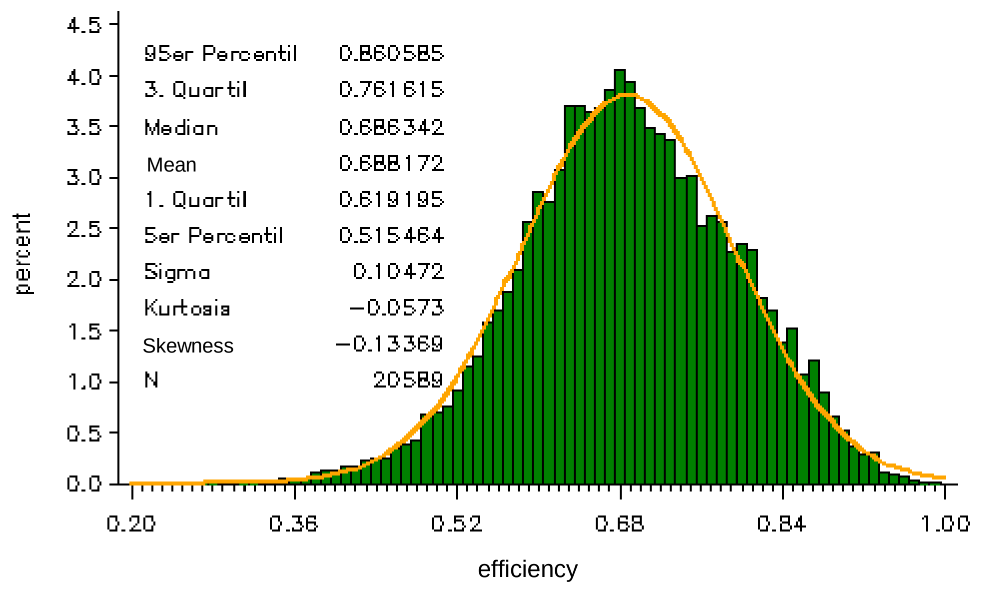
[13] Here the bootstrapping procedure was applied with a B of 2000.

possible yearly combinations it is most reasonable to use 1995 for t and 2004 for $t+1$. If there was a change in technology over time the larger the time spread the clearer should be the result. The results are presented in Figure 2.

As the Figure reveals, there was a change in technology over time. Even with a mean of 1.027, at least 50 percent of the FSE's take values grater 1.07 or less than 0.97. Reminding the effects that intersecting frontiers do have on the FSE, the results above indicate that a common frontier is no option. In fact the following calculations will be based on yearly frontiers. This also accounts for the fact that the consecutive observations for round about fifty percent of all firms within the sample lie within a timeframe of two years.

In doing so, the first Hypothesis concerning the efficiency of engineering firms can be tested. It was presumed to find a majority of firms operating quite efficiently, meaning that they ought to have efficiency scores of one or at least near one. Moreover, it was expected to find a density that is skewed to left, since there should be more efficiency scores above the mean (Hypothesis 2). This would fit with the basic assumption that only efficient firms survive in a competitive environment and stay in the market, while less efficient firms will exit the market sooner or later.

Figure 3: Technical efficiency[14]



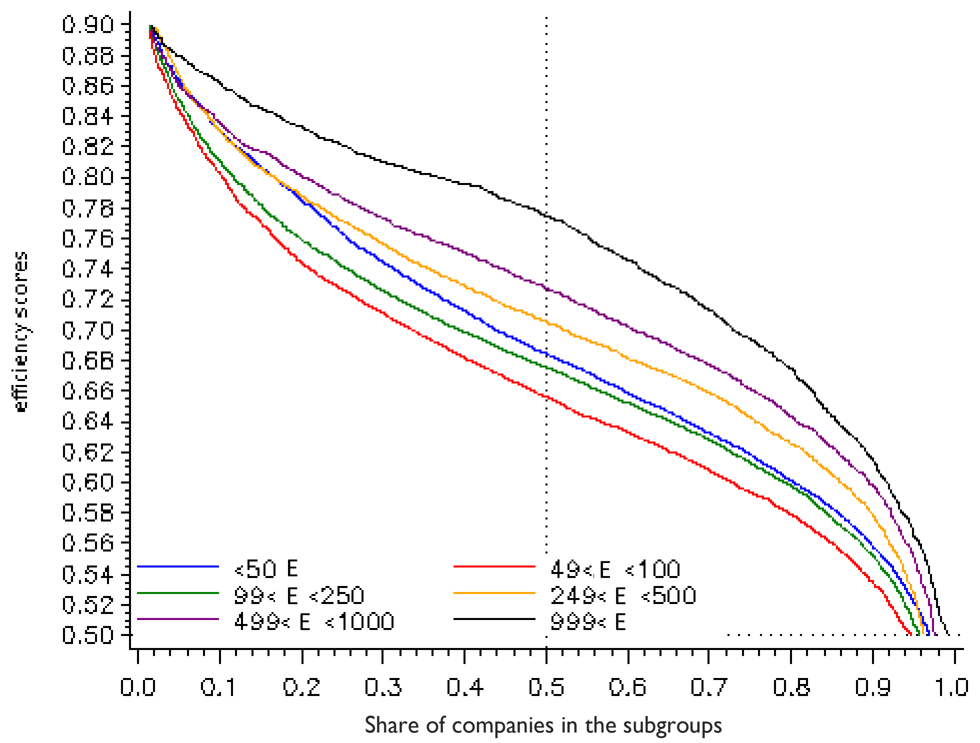
The actual estimates reject the first hypothesis. As Figure 3 brought to light the majority of the firms is working at an efficiency level far below one. The mean as well as the median is roughly 0.69. Hence, the engineering firms are operating inefficiently on average, using about 31 percent more input than necessary. Furthermore in fifty percent of all observations the firms do work at an efficiency level of 0.62 to 0.76, meaning that they use 24

14 The figure contains all estimated efficiency scores for all years. The estimation was done using the modified bootstrap procedure as proposed by Simar and Wilson (1998, 2000a, 2000b) utilizing a B of 2000.

to 38 percent more input than necessary. Moreover, in just five percent of all cases firms have had an efficiency score above 0.86. In contrast, 20 percent of all observed efficiencies are between 0.62 and 0.52. Hence, in these cases firms used almost twice as much input as necessary.

Even the form of the density is surprising. The estimated efficiency scores have an almost normal distributed density (with $\mu = 06883$ and $\sigma = 0{,}105$).[15] So, there are almost as many firms with efficiency scores below the mean as there are firms with an efficiency score above the mean. Eventually the second hypothesis of a density that is skewed to the left is not supported by the results either.

Figure 4: Distribution of efficiency per size-class [16]



In order to get to a more differentiated picture the firms were separated by size using the number of employees as criterion[17], since engineering firms are characterized by small-lot production or even single-part production (Wiechers and Schneider, 2005). Hence, the number of employees is highly related to the output of a firm. The presentation of the firm's efficiency scores takes place using a kind of reverse distribution function as proposed by Fritsch and Stephan (2003). Looking at Figure 4 it appears that there are significant differences

[15] A test of normal distribution rejects the null hypothesis. The statement of "almost" just addresses the optical conformity between the probability density and the diagrammed density of the normal distribution (yellow line). The test results are presented in Table 7, Appendix A.

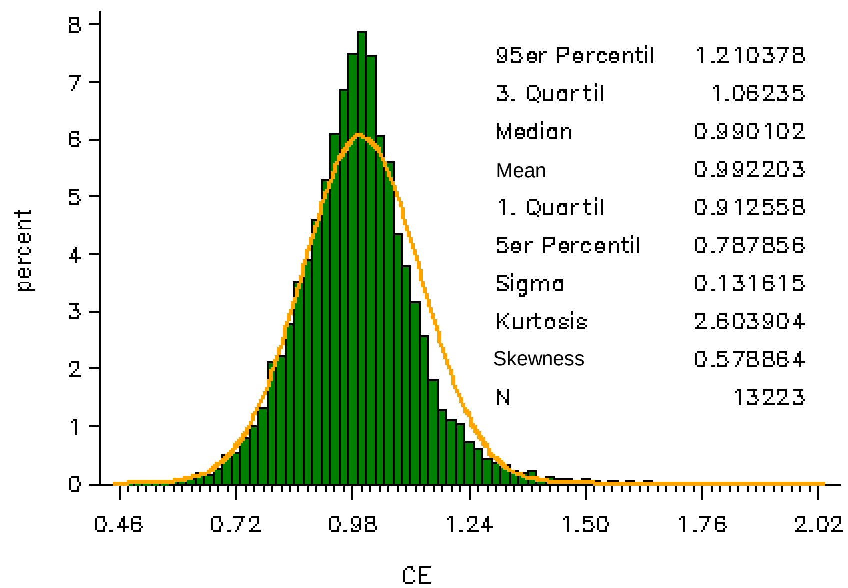
[16] To get a better impression the diagramed is limited to efficiency scores between 0.9 and 0.5. Of course the shares of the firms are calculated using all estimates.

[17] The classes are defined as follow: 1-49 employees (E), 50-99 E, 100-249 E, 250-499 E, 500-999 E and 1000 and more employees.

between the groups.[18] For instance the median company in the class of 50 – 99 employees (red line) has an efficiency of just 65.62 percent, while the median company in the class of the biggest companies (black line) has an efficiency score of 77.52 percent. So, the latter works much more efficient than the former. A reason for this difference could be the fact that larger companies acting globally face a higher competition. Moreover it's argued that they better penetrate the market and have more funds to employ a better management (Badunenko, 2007).

Beyond that finding another surprising fact can be revealed by looking at the blue line. The line shows the efficiency scores of the smallest companies. As it is obvious, it lies above the lines (green and red) of the next classes (50 – 99 E and 100 – 250 E). It becomes apparent that small companies are on average more efficient than the next larger ones. The reasons for that are unknown, but one could speculate that there is some kind of critical mass. That is, the growth of a company goes along with a decline in efficiency. It is reasonable to assume that the price of growth is a decline in efficiency. This happens until some critical mass is achieved and the firms are forced and have the ability to enhance their efficiency in order to survive in a global market, or to fall back and reduce size again (or even exit the market). This would fit with the observation that the firms in each successive class greater than the class of 100 – 249 E are on average more efficient than their predecessor.

Figure 5: The catching –up effect over the whole period[19]



Besides the actual efficiency its change over time is also of interest. It was argued that the change in efficiency should be rather small, if firms should operate efficiently. It is unclear to what extend this expectation of

18 The ANOVA test by Welch confirms that the observed differences are significant. See for results Table 9 in Appendix A.

19 The estimation was done using the modified bootstrap procedure utilizing a B of 2000

small changes still holds, as the actual efficiency scores are found to be almost normal distributed. As defined above the tool to measure the efficiency over time will be the catching-up effect (CE). Moreover, the applied time frame will be the yearly change in order to use as much of the available data as possible.[20] The following Figure 5 shows all CE – values for all size-classes and all yearly intervals.

First of all it seems that the assumption of small changes still holds. The mean as well as the median is almost one (0.99), i.e. on average there have been no great changes in efficiency over time (on a yearly basis). Nevertheless the figure shows a rather large standard deviation of 0.1316. Based on normal distribution 68 percent of all observed changes would be in the range of 0.99 ± 0.1316. Admittedly the CE – data have no normal distribution (with $\mu = 0.99$ and $\sigma = 0{,}1316$) as it is diagrammed as yellow line in Figure 5.[21]

Table 4: Share of observed CE – values of different size (percent)

CE<0.9	0.9≤CE<0.95	0.95≤CE≤1.05	1.05<CE≤1.1	1.1<CE
21.92	13.68	35.76	11.98	16.66

However, as Table 4 reveals in 35.76 percent of all observations the change was below five percent. On the other hand, there are yearly changes in efficiency of 5 to 10 percent in 25.66 percent of all measured changes. Moreover, there have been changes above 10 percent. Thus, a reduction in efficiency of more than 10 percent could be observed in 21.92 percent of all changes. In contrast an increase in efficiency by more than 10 percent took place in 16.66 percent of all cases. So, even if one has to admit that the mean as well as the median is close to one, more than one third of all observed changes are above ten percent. The data also reveal that the share of observations with a decrease in efficiency prevails. This is not surprising, since an increase in efficiency is linked with some efforts by the company while a decrease is not.

The results above lead to the question whether or not changes in efficiency are also significantly different if separated by firm size. Since one of the assumptions of the ANOVA does not hold in this context, the Wilcoxon rank-sum test is applied.[22] As the upper part of Table 5 reveals at a significance level of 5 percent the null hypothesis of identical distributed efficiency changes often cannot be rejected. Thus, for instance the efficiency changes in the class of 100 – 249 employees are not (significantly) different compared to the efficiency changes in the class of 50 – 99 employees. Moreover as the lower part of Table 5 shows, even in cases where the null hypothesis is rejected the actual estimates show only minor differences. A graphical analysis confirms that there are differences among the classes but that these differences are negligible.[23] Thus it seems reasonable to state that the size of a firm has no big impact if it comes to yearly efficiency changes, although there is evidence that size is not entirely nonrelevant.

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20 This accounts for the fact, that almost fifty percent of the firms are part of the sample with just two observations in consecutive years. See again Table 2.

21 The test results of the test of normality are reported in Table 10 in Appendix A.

22 It's the assumption of homogenous residuals.

23 For an optical impression see Figure 7 in Appendix A.

Table 5: Test and descriptive statistic of CE – values per size class[24]

	size–classes					
	49	**99**	**249**	**499**	**999**	**1000**
all	<0.01	0.6492	0.8050	0.0370	<0.01	<0.01
49		<0.01	<0.01	<0.01	<0.01	<0.01
99			0.5812	0.039	<0.01	<0.01
249				0.0986	<0.01	<0.01
499					<0.01	<0.01
999						1
N	2950	2622	2924	1481	975	745
99er percentile	1.3543	1.3947	1.3715	1.3697	1.3300	1.4108
3. quartile	1.0524	1.0646	1.0606	1.0617	1.0757	1.0607
median	0.9733	0.9846	0.9887	0.9925	1.0063	0.9989
mean	0.9781	0.9899	0.9906	0.9968	1.0049	0.9989
1. quartile	0.8930	0.9033	0.9093	0.9262	0.9353	0.9389
1er percentile	0.6956	0.6781	0.7053	0.7039	0.6912	0.6709
sigma	0.1345	0.1401	0.1297	0.1284	0.1199	0.1291

We also tested whether there is a time related factor that has an influence on the change of efficiency. Therefore, the estimates are separated by periods and the Wilcoxon rang-sum test is once again used to determine whether or not the efficiency changes are significantly different in each period. In the actual analysis the periods 96-97, 98-99 and 02-03 have been excluded due to the fact that the number of firms which had reported their data in both years is relatively small. This traces back to the recollection of data in 1997, 1999 and 2003.

As Table 6 reveals, at a significance level of 5 percent the null hypothesis of no differences between the periods cannot be rejected in just two cases. Thus, the changes between 1995 and 1996 are not that different from the changes in period 1997 to 1998. Furthermore, the changes in efficiency in period 95-96 are not significantly different to the changes in period 00-01. On the other hand it is quite obvious that the change in efficiency in all other periods is significantly different. Thus, the mean in period 99-00 shows, that the efficiency decreases by roughly 8.5 percent on average, whereas in period 01-02 the firms enhanced their efficiency by 4 percent on average. Comparing the results as presented in Table 5 and 6 it becomes apparent, that some time related component is of importance in explaining the change of efficiency rather than the size of a firm.

It seems reasonable to expect this component to be the engineering business cycle. The idea is the following: In rather recessive years (compared to the previous) the companies efforts to reduce inputs in order to account for a decreasing output could be not strong enough. Thus, in the context of the DEA model the effect should be a reduced efficiency and accordingly a CE below one. On the other hand if there is a cyclical good year, companies usually increase their outputs. If this goes along with an almost stable input or at least with an

[24] The figures in the header denote the size classes by using the upper boundary as name, e.g. the class of 0 top 49 employees is named 49. The same applies to the row names in the upper part of the table.

input that does not increase as much as the output the efficiency of a firm should also rise on average and hence the CE should be above one.

Table 6: Test and descriptive statistic of CE – values per time frame

	periods					
	95-96	**97-98**	**99-00**	**00-01**	**01-02**	**03-04**
all	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
95-96		0.2393	<0.01	0.0555	<0.01	<0.01
97-98			<0.01	<0.01	<0.01	<0.01
99-00				<0.01	<0.01	<0.01
00-01					<0.01	<0.01
01-02						<0.01
N	1980	1851	1977	1930	1836	2123
99er percentile	1.3688	1.3697	1.2749	1.3739	1.4922	1.3431
3. quartile	1.0707	1.0678	0.9818	1.0704	1.1085	1.0452
median	1.0037	1.0040	0.9095	0.9976	1.0312	0.9705
mean	1.0074	1.0111	0.9149	0.9967	1.0402	0.9747
1. quartile	0.9320	0.9472	0.8412	0.9229	0.9599	0.8911
1er percentile	0.7386	0.7033	0.6382	0.7076	0.7269	0.6998
sigma	0.1228	0.1232	0.12	0.1282	0.1393	0.1267

In order to see whether this expectation holds the cyclical development in the engineering branch, measured by the production index[25], is compared with the changes in efficiency measured by the CE. Furthermore the data are not only separated by periods but also by the size of the firms. According to the results as presented in Table 5, the size classes are now defined as follows: 0 – 49, 50 – 249, 250 – 499 employees and firms with 500 or more employees.

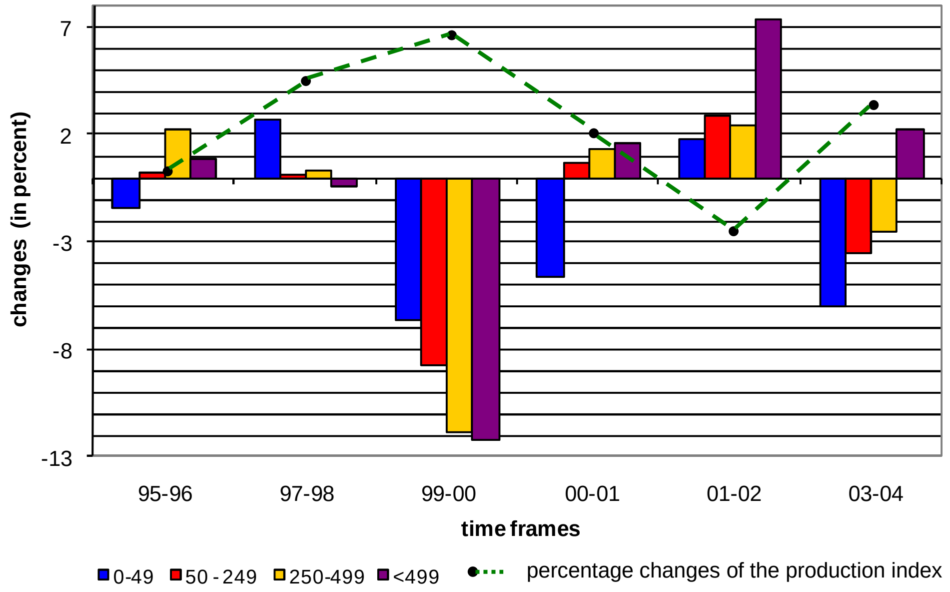
Figure 6 reveals that the relationship between efficiency changes and the economic cycle does not have the supposed form. Looking at the first two periods we see an economic upswing while the efficiency changes only slightly and non-uniform. In contrast the economic upswing is quite strong between 1999 and 2000 while the efficiency is now dramatically decreasing. Looking at period 01-02 roughly the same is observable, of course with an opposite sign.

Thus, it can be concluded that the relation between the change of efficiency and the business cycle is not as supposed above. In fact it seems that efficiency is of no relevance in the management objectives if the overall economic situation is good or actually perfect. The impression is that firms (on average) simply employ the input that they think is necessary to meet the sold output without looking much at efficiency. On the other hand, during an economic slowdown enhancing the efficiency seems to become important, resulting in cost reduction programs or something alike. To stress this finding one has to remember the way the efficiency is calculated and the fact that the efficiency for each firm in each year is calculated at a frontier of the corresponding year.

[25] The according time series were taken from the Genesis Data Base of the German Federal Statistic office.

Thus, the decrease in input during an economic slowdown must be stronger than the decrease of output. In contrast, an increasing output goes along (on average) with an increasing amount of input and the latter increase needs to be stronger than the former. It seems reasonable to characterize the relation between efficiency change and the economic trend as follows: On average the efforts to enhance efficiency or even to hold it constant are reduced in cyclical good years, which results in decreasing efficiency et vice versa. As the results in period 03-04 as well as in period 00-01 shows, there need to be other factors to explain the heterogeneous findings in the mentioned periods.

Figure 6: Median CE – values broken down into years and size classes and the engineering business cycle[26]



V. Conclusion

This paper investigates the efficiency of German engineering firms. According to the success of these firms on many markets and their output growth two expectations were postulated, namely first of all that the observed success is associated with an efficient use of inputs in the production process, where the majority of the engineering firms works highly efficient. Secondly as there are always firms that perform rather poorly, it seemed reasonable to expect the density of the measured efficiency to be skewed to the left. If these expectations should hold, it would also be reasonable to expect that the efficiency changes are rather small.

[26] The changes as diagramed are calculated as follows: CE•100-100. Thus, the median change in the class of 0 to 49 employees say's that the efficiency dropped by 1.39 percent between 1995 and 1996. Accordingly the actual CE before its conversion was 0.9861.

The data envelopment analysis was used to test these expectations, whereas the Farrell's efficiency scores are estimated. The analysis was conducted based on a sample of 20,589 observations for the period 1995 to 2004. The first task was to determine which assumption concerning the economies of scale hold. Using the Kolmogorov-Smirnov and the Wilcoxon rank-sum test it was found that the frontier is determined by variable returns to scale. Based on this finding the next question concerning the utilization of a common frontier was analyzed. According to the results the use of a common frontier is no option in the present analysis since the technology has changed over time. Unfortunately the FSE, which was used to detect this change, loose some of its explanatory power if the frontier is defined by more than two inputs. The same is true if the frontier is characterized by variable returns to scale. Since both qualities are true in the present context no concrete statement concerning the modality of the frontier shift and consequently the technological change is possible. However, the results leave no doubt that there was a change.

After making sure that the assumptions underlying the model are correctly specified the actual analysis was conducted going after the first expectation. The efficiency scores have been estimated using the appropriate bootstrap and employing a yearly frontier. The results are surprising. First of all it has to be stated that the majority of the firms is operating inefficient. The mean of all estimates over the whole period is roughly 0.69. Hence, the engineering firms could, on average, reduce their input by 31 percent while producing the same output. The expectation that the success of the German engineering firms goes along with an efficient use of inputs does not hold. Another interesting finding is the density of the efficiency scores. It seemed reasonable to assume that the density would be skewed to the left. The analysis revealed that this expectation does not hold either. The actual estimates are not normal distributed in a statistical sense, i.e. the test for normality did not support such a predication. Nevertheless the probability density of the efficiency scores is almost normally distributed as can be seen in Figure 3.

To take a closer look at the efficiency scores the companies were also separated by size. The efficiency scores in each of the classes differ significantly. Hence it can be pointed out that size is of importance for efficiency. Of course the argument could be brought forward that it is the efficiency that is important for size, i.e. companies will only become larger if they are efficient. First of all the results show, that there are highly efficient companies as well as highly inefficient companies in each class. Moreover, it was shown that, in the beginning and on average, the efficiency falls if firms become larger, so that the size influences efficiency rather than efficiency influences size. The results furthermore lead to the expectation that growth usually goes along with a decline of efficiency, or in other words that the price of growth is a decrease in efficiency. A test whether this growth/price relation holds until some critical mass is achieved enabling the firms to enhance their efficiency once again is left for further research.

Besides efficiency the change of efficiency over time was also of interest. It was expected that the change will be rather small if the majority of the firms should work efficiently. This expectation was rejected as well. Even though the mean as well as the median was found to be very close to one, roughly 38 percent of the yearly changes were above ten percent. Since the differences between the periods are found to be significantly in almost all cases it seems reasonable that a factor that drives changes in efficiency would be the engineering business cycle. The idea behind this expectation is based on the models approach to measure inefficiency. If the output is reduced while the input is not, accordingly inefficiency should increase et vice versa. The reality behind this expectation could be that most firms do not react quickly enough when the output goes down by reducing the amount of employed inputs (e.g. labor input) accordingly. On the other hand if the overall economic situation becomes better and firms are able to increase their output the additional input required (again for example labor) is already employed and therefore the efficiency increases almost automatically. The actual findings lead to doubts concerning such coherences. In fact it seems that firms neglect their efficiency, simply producing the desired output if the overall economic situation is good, resulting in an increasing inefficiency. In contrast to this if the overall economic situation becomes rather difficult, compared to the previous year, the companies put more emphasize on efficiency and they reduce the input at a degree that is stronger than the decline of output. Of course this finding needs further research in order to be more than just a first observation.

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Appendix A

Table 7: Test for normal distributed efficiency scores[27]

test	statistics		p-value	
Kolmogorov -Smirnov	D	0.0148	Pr > D	<0.01
Anderson-Darling	A-Sq	6.8339	Pr > A-Sq	<0.01

Table 8: Frequency of firm observations in the sample

year	number of observations	Share of all observations (percent)	cumulative number of observations	cumulative share of all observations (percent)
1995	2023	9.83	2023	9.83
1996	2064	10.02	4087	19.85
1997	1931	9.38	6018	29.23
1998	1932	9.38	7950	38.61
1999	2108	10.24	10058	48.85
2000	2146	10.42	12204	59.27
2001	2051	9.96	14255	69.24
2002	1934	9.39	16189	78.63
2003	2223	10.80	18412	89.43
2004	2177	10.57	20589	100.00
Total	20,589	100		

Table 9: ANOVA for efficiency and size classes

	ANOVA by Welch		
	degrees of freedom	F-statistic	Pr > F
groesse[28]	5	237.6	<.0001
error	6493.4		

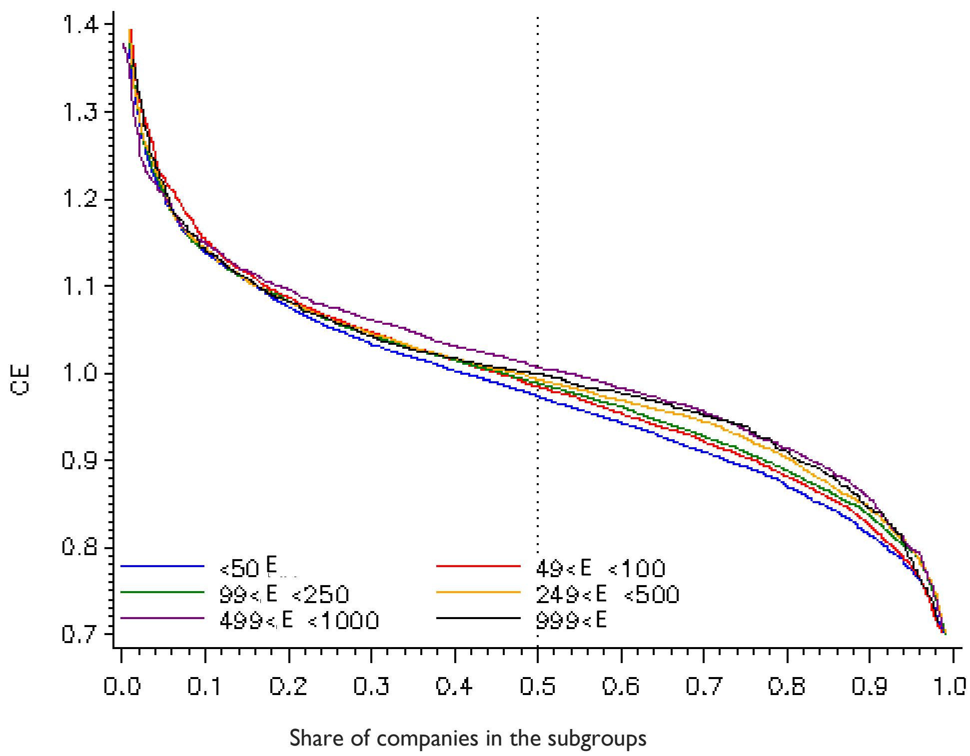
27 The null hypothesis postulates the efficiency scores to be normal distributed. For more information about the Anderson-Darling test see Schlittgen and Streitberg (1997).

28 German name of the variable that separates the efficiency scores by the size of the firms.

Table 10: Test for normal distributed CE - values

test		statistic	p-value	
Kolmogorov -Smirnov	D	0.0474	Pr > D	<0.01
Anderson-Darling	A-Sq	54.548	Pr > A-Sq	<0.01

Figure 7: Distribution of the CE-values per size-class

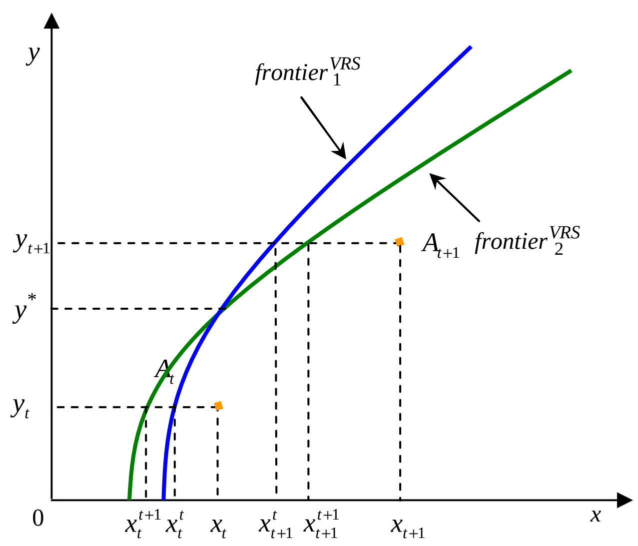


Appendix B

Suppose for simplicity a branch which is operating with just one input and one output. Furthermore assume (also for simplicity) that the true production set is known. Consequently the true frontier and the economies of scale in that branch are known as well. For our purpose the frontier ought to exhibit variable returns to scale. Moreover suppose that observations out of two points in time, $t, t+1$ (e.g $t = 1$ and $t + 1 = 2$) are available. Over time a change in technology should have occurred that takes the form as diagramed below. Hence, smaller companies have gained since the need fewer inputs in $t + 1$ to produce the same amount of output as in t (i.e. if the produce outputs below y^*). In contrast large companies (i.e. with output levels above y^*) now need to assemble more inputs in order to produce the same output in $t + 1$ as in t.

First of all assume that the output of firms hasn't changed much between the two point in times so that the output will either be $y_{it} < y^*$ or $y_{it} > y^*$ with $t = 1,2$. Hence, the FSE will take values below one (for all firms with $y_{it} > y^*$) as well as above one (for all firms with $y_{it} < y^*$). So, the FSE allows no distinct conclusion concerning the direction of the technology shift (of course in this example it's possible since a graph is available, if it comes to four or more dimensions graphical analysis is no longer possible).

Figure 8: FSE and variable returns to scale



Now consider there is a company A that produces at the point in time t an output of y_t using an input quantity of x_t. For our purpose the output of y_t should be below y* as it is diagramed above. Over time this company became larger and at point in time $t + 1$, it assembles an input of x_{t+1} in order to produce an output of y_{t+1}. This output shall be above y*. Thus, the situation is as diagrammed above.

Firstly it became obvious that the inefficiency of firm A at each point in time has no influence regarding the value of its FSE. This traces back to the fact that the observed output/input relation in each component in the brackets is kept constant.[29] Thus, just the location of both frontiers for each output level is of importance for the value of the FSE. If one defines the first component in the brackets as $FSEa = (\theta_t(x_{t+1}, y_{t+1})/\theta_{t+1}(x_{t+1}, y_{t+1}))$ and the second component as $FSEb = (\theta_t(x_t, y_t)/\theta_{t+1}(x_t, y_t))$ the direction of the frontier shift as it is revealed by the FSE depends on the successive basic constraints:

1. $FSEb > 1/FSEa \rightarrow FSE > 1$
2. $FSEb < 1/FSEa \rightarrow FSE < 1$
3. $FSEb = 1/FSEa \rightarrow FSE = 1$

Hence, the FSE will only takes values greater one if the second frontier lies enough to left of the first one (relatively) for one of the two output levels. Secondly, it is important to notice that the FSE will tend to one even if there was a significant change in technology since either the FSEa or the FSEb will take a value greater one while the other will be below one. In the extreme the third constraint is fulfilled and the FSE will be one. Furthermore, since the product of the FSEa and the FSEb is treated with a root, the FSE must tend towards one. In the end the FSE can take values near one even if there was a significant change in technology provided that the frontiers intersect.

[29] The first component in the brackets is $\theta_t(x_{t+1}, y_{t+1})/\theta_{t+1}(x_{t+1}, y_{t+1})$. In the two dimensional example above this could be rewritten as $\theta_t(x_{t+1}, y_{t+1}) = 0x_{t+1}^{t}/0x_{t+1}$ and $\theta_{t+1}(x_{t+1}, y_{t+1}) = 0x_{t+1}^{t+1}/0x_{t+1}$. It follows $(0x_{t+1}^{t}/0x_{t+1})/(0x_{t+1}^{t+1}/0x_{t+1})$ and therefore $0x_{t+1}^{t}/0x_{t+1}^{t+1}$. The same can be shown for the second component in the brackets. In the end the second component can be reduced to $0x_t^{t}/0x_t^{t+1}$. As it is obvious, the outcome of the FSE depends only upon the location of the two frontiers at each point in time and for each output level.

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